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The Polarized Structure Function g_2: A Lattice Study Revisited*

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Abstract

A recent lattice calculation of the spin-dependent structure function g_2 is revisited. It has been recognized that the twist-three operator, which gives rise to d_2, mixes non-perturbatively with operators of lower dimensions under renormalization. This changes the results substantially.

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1 Introduction

The nucleon's second spin-dependent structure function g_2 is of considerable phenomenological interest. The discussion follows the operator product expansion (OPE) [1]. In leading order of $1/Q^2$, g_2 receives contributions from both, twist-two and twist-three operators. It thus offers the unique possibility of directly assessing higher-twist effects. The twist-three operator probes the transverse momentum distribution of the quarks in the nucleon, and has no simple parton model interpretation.

A few years ago we have computed the lowest non-trivial moment of g_2 on the lattice [2]. To convert the lattice numbers to continuum results, the operators entering the OPE have to be renormalized. An essential feature of our calculation was that the renormalization was done in perturbation theory. It turned out that the twist-three contribution was the dominant contribution to both, the proton and the neutron structure functions.

Renormalization effects are a major source of systematic error. In this talk we shall extend our previous work by employing non-perturbative renormalization. The novel feature of this approach is that it introduces mixing with lower-dimensional operators.

2 OPE and Mixing

In leading order of $1/Q^2$, and for massless quarks, the moments of g_2 are given by

$$2\int_0^1 \mathrm{d}x x^n g_2(x,Q^2) = \frac{1}{2}\frac{n}{n+1}\sum_{f=u,d}\big[e_{2,n}^{(f)}(\mu^2/Q^2, g(\mu))\, d_n^{(f)}(\mu) - e_{1,n}^{(f)}(\mu^2/Q^2, g(\mu))\, a_n^{(f)}(\mu)\big] \tag{1}$$

for even $n \geq 2$, where [1]

$$\langle \vec{p},\vec{s}|\mathcal{O}^{5(f)}_{\{\sigma\mu_1\cdots\mu_n\}}|\vec{p},\vec{s}\rangle = \frac{1}{n+1}a_n^{(f)}\,[s_\sigma p_{\mu_1}\cdots p_{\mu_n} + \cdots - \text{traces}], \tag{2}$$

$$\langle \vec{p},\vec{s}|\mathcal{O}^{5(f)}_{[\sigma\{\mu_1]\cdots\mu_n\}}|\vec{p},\vec{s}\rangle = \frac{1}{n+1}d_n^{(f)}\,[(s_\sigma p_{\mu_1} - s_{\mu_1}p_\sigma)p_{\mu_2}\cdots p_{\mu_n} + \cdots - \text{traces}], \tag{3}$$

$$\mathcal{O}^{5(f)}_{\sigma\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^n \bar{\psi}\gamma_\sigma\gamma_5 \overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n}\psi - \text{traces}, \tag{4}$$

and $e_{1,n}^{(f)}$, $e_{2,n}^{(f)}$ are the Wilson coefficients. Here $\{\cdots\}$ ($[\cdots]$) means symmetrization (antisymmetrization). The operator (2) has twist two, whereas the operator (3) has twist three. Both, the Wilson coefficients and the operators are renormalized at the scale μ. It is assumed that the Wilson coefficients can be computed perturbatively, whereas the calculation of the reduced matrix elements $a_n^{(f)}$ and $d_n^{(f)}$ is a problem for the lattice. In the following we shall drop the flavor indices, unless they are necessary.

The lattice calculation splits into two separate tasks. The first task is to compute the nucleon matrix elements of the appropriate lattice operators. This was described in detail in [2]. The second task is to renormalize the operators. Generically,

$$\mathcal{O}(\mu) = Z_{\mathcal{O}}((a\mu)^2, g(a))\, \mathcal{O}(a). \tag{5}$$

As in the continuum, we impose the (MOM-like) renormalization condition

$$\tfrac{1}{4}\,\mathrm{Tr}\,\langle q(p)|\mathcal{O}(\mu)|q(p)\rangle \Big[\langle q(p)|\mathcal{O}(a)|q(p)\rangle\,|^{\mathrm{tree}}\Big]^{-1} \underset{p^2=\mu^2}{=} 1, \tag{6}$$

where $|q(p)\rangle$ is a quark state of momentum p in Landau gauge. In our earlier work [2, 3] we have computed the renormalization constants in perturbation theory to one-loop order. A restriction of the perturbative calculation is that it does not allow to incorporate mixing with lower-dimensional operators.

In a recent paper [4] we have started a non-perturbative calculation of the renormalization constants associated with the structure functions F_1, F_2 and g_1. Let us here consider the case of the structure function g_2. We shall restrict ourselves to $n = 2$. This is the lowest moment of g_2 for which the OPE makes a statement. As before [2], we take $\sigma = 2$, $\mu_1 = 1$ and $\mu_2 = 4$. We thus need to consider the operators

$$\bar{\mathcal{O}}^5_{\{214\}} =: \mathcal{O}^{\{5\}} \tag{7}$$

and

$$\begin{aligned}
\bar{\mathcal{O}}^5_{[2\{1]4\}} &= 2\bar{\mathcal{O}}^5_{2\{14\}} - \bar{\mathcal{O}}^5_{1\{24\}} - \bar{\mathcal{O}}^5_{4\{12\}} \\
&= \bar{\psi}\Big(\gamma_2 \overleftrightarrow{D}_1 \overleftrightarrow{D}_4 + \gamma_2 \overleftrightarrow{D}_4 \overleftrightarrow{D}_1 - \tfrac{1}{2}\gamma_1 \overleftrightarrow{D}_2 \overleftrightarrow{D}_4 - \tfrac{1}{2}\gamma_1 \overleftrightarrow{D}_4 \overleftrightarrow{D}_2 \\
&\qquad -\tfrac{1}{2}\gamma_4 \overleftrightarrow{D}_1 \overleftrightarrow{D}_2 - \tfrac{1}{2}\gamma_4 \overleftrightarrow{D}_2 \overleftrightarrow{D}_1\Big)\gamma_5\psi \\
&=: \mathcal{O}^{[5]},
\end{aligned} \tag{8}$$

with $\bar{\mathcal{O}}^5$ indicating the Euclidean counterpart of $\mathcal{O}^5$, which belong to the representation $\tau_3^{(4)}$ and $\tau_1^{(8)}$, respectively, of the hypercubic group $H(4)$ [5]. The operator (8) has dimension five and C-parity +. It turns out that there exist two operators of dimension four and five, respectively, transforming identically under $H(4)$ and having the same C-parity, with which (8) can mix:

$$\mathrm{i}\,\bar{\psi}\Big(\sigma_{13} \overleftrightarrow{D}_1 - \sigma_{43} \overleftrightarrow{D}_4\Big)\psi =: \mathcal{O}^{\sigma}, \tag{9}$$

$$\bar{\psi}\Big(\gamma_1 \overleftrightarrow{D}_3 \overleftrightarrow{D}_1 - \gamma_1 \overleftrightarrow{D}_1 \overleftrightarrow{D}_3 - \gamma_4 \overleftrightarrow{D}_3 \overleftrightarrow{D}_4 + \gamma_4 \overleftrightarrow{D}_4 \overleftrightarrow{D}_3\Big)\psi =: \mathcal{O}^0. \tag{10}$$

The operator (10) vanishes in tree approximation between quark states. It therefore cannot be included in our framework, and so we discard it. We then remain with

$$\mathcal{O}^{[5]}(\mu) = Z^{[5]}(a\mu)\mathcal{O}^{[5]}(a) + Z^{\sigma}(a\mu)\mathcal{O}^{\sigma}(a). \tag{11}$$

The renormalization constant $Z^{[5]}$ and the mixing coefficient Z^{σ} are determined from

$$\tfrac{1}{4}\,\mathrm{Tr}\,\langle q(p)|\mathcal{O}^{[5]}(\mu)|q(p)\rangle\Big[\langle q(p)|\mathcal{O}^{[5]}(a)|q(p)\rangle\,|^{\mathrm{tree}}\Big]^{-1} \underset{p^2=\mu^2}{=} 1, \tag{12}$$

$$\tfrac{1}{4}\,\mathrm{Tr}\,\langle q(p)|\mathcal{O}^{[5]}(\mu)|q(p)\rangle\Big[\langle q(p)|\mathcal{O}^{\sigma}(a)|q(p)\rangle\,|^{\mathrm{tree}}\Big]^{-1} \underset{p^2=\mu^2}{=} 0. \tag{13}$$

Let us now see how much this effect changes our results for d_2.

3 Results and Conclusions

We work on a $16^3\,32$ lattice at $\beta = 6.0$ using quenched Wilson fermions. The lattice spacing is $a^{-1} \approx 1.95\,\mathrm{GeV}$. The calculations are done at three values of the hopping parameter κ, so that we can extrapolate our results to the chiral limit. We denote the reduced matrix elements of the unrenormalized operators $\mathcal{O}^{[5]}$ and $\mathcal{O}^{\sigma}$ by $d_2^{[5]}$ and d_2^{σ}, respectively. In table 1 we give our results. The numbers for a_2 and $d_2^{[5]}$ (formerly called d_2) are taken over from [2]. These numbers are based on $O(400-1000)$ configurations, depending on the value of κ. The results for d_2^{σ} are new, and they are based on $O(80)$ configurations only. Hence the relatively large statistical error.

The calculation of the renormalization constants follows [4], supplemented by eqs. (11) - (13). We denote the renormalization constant of the operator $\mathcal{O}^{\{5\}}$ by $Z^{\{5\}}$. In figs. 1 - 3 we plot the renormalization constants as a function of μ^2. The numbers given refer to the MOM scheme, and they have been extrapolated to the chiral limit. We fit $Z^{\{5\}}$ and $Z^{[5]}$ by $A + B\ln(\mu) + C/\mu$, and Z^{σ} by $A + C/\mu$. The result of the fit is shown by the solid lines. In order to match our results with the perturbatively known Wilson coefficients [6], we need to transform the renormalization constants to the $\overline{\mathrm{MS}}$ scheme. This is done to one-loop order [3].

Operator	κ			
	0.1515	0.153	0.155	$\kappa_c = 0.1569$
$a_2^{(u)}$	0.146(9)	0.142(9)	0.138(17)	0.132(23)
$a_2^{(d)}$	-0.032(3)	-0.032(3)	-0.044(9)	-0.043(10)
$d_2^{[5]\,(u)}$	-0.097(5)	-0.122(6)	-0.177(14)	-0.206(18)
$d_2^{[5]\,(d)}$	0.018(2)	0.021(2)	0.032(5)	0.035(6)
$d_2^{\sigma\,(u)}$	-0.74(12)	-0.89(20)	-1.27(110)	-1.31(69)
$d_2^{\sigma\,(d)}$	0.136(26)	0.143(42)	0.116(119)	0.137(122)

Table 1: The unrenormalized, reduced matrix elements a_2, $d_2^{[5]}$ and d_2^{σ} for u and d quarks separately.

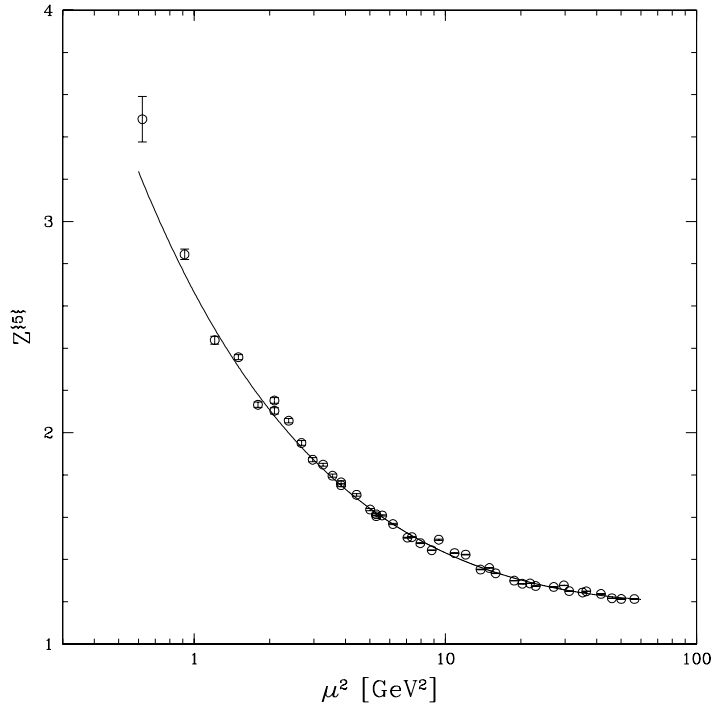

Figure 1: The renormalization constant $Z^{\{5\}}$.

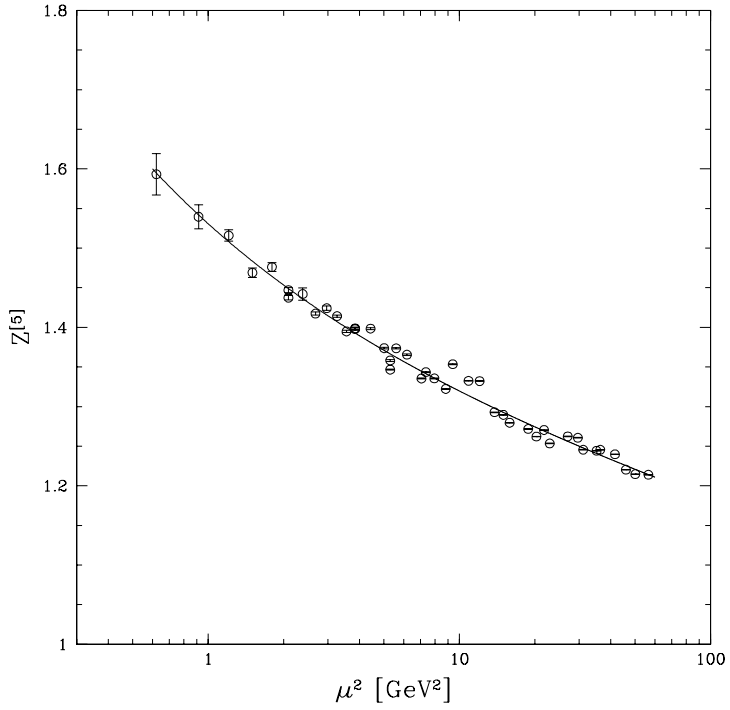

Figure 2: The renormalization constant $Z^{[5]}$.

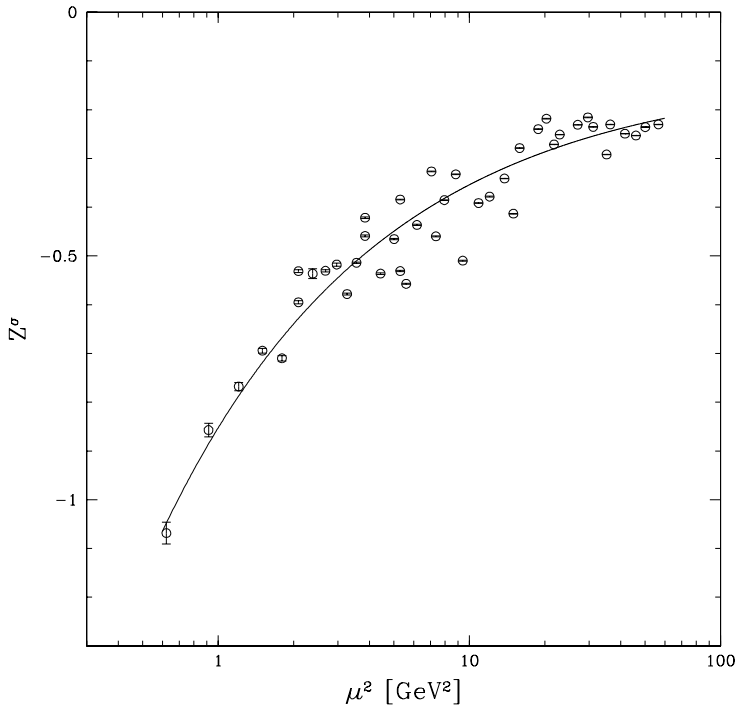

Figure 3: The renormalization constant Z^σ.

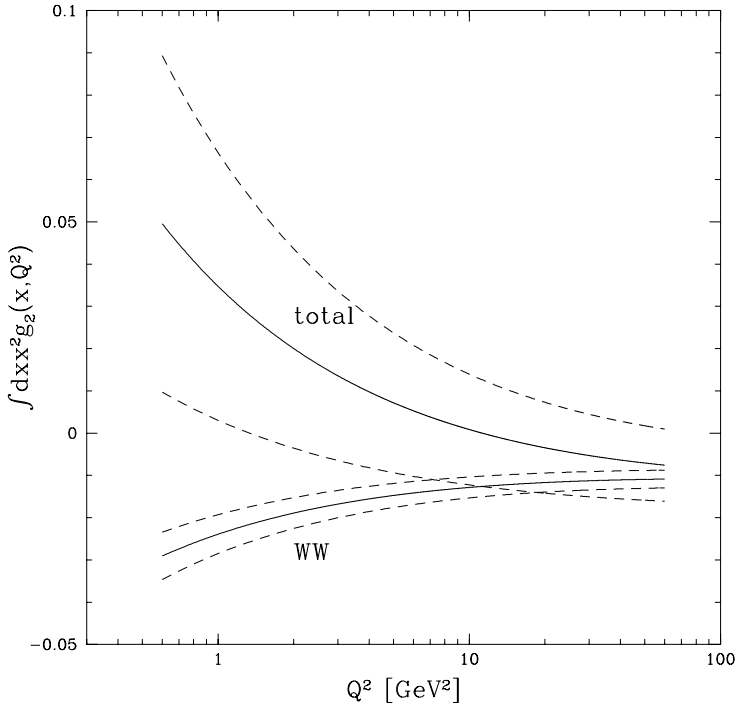

Figure 4: The lowest moment of $g_2(x, Q^2)$ for the proton as a function of Q^2.

We are now ready to state our results for g_2. At larger values of μ^2 ($\mu^2 \gtrsim 7\,\mathrm{GeV}^2$) the μ–dependence of $Z^{\{5\}}$ and $Z^{[5]}$ is reasonably well described by the one-loop perturbative result (see also [4]), so that we may take $\mu^2 = Q^2$. This choice should keep higher-loop corrections to the Wilson coefficients small. In fig. 4 we show our new results for the proton. The curve marked 'total' gives the total contribution of both, twist-two and twist-three matrix elements. The curve marked 'WW' corresponds to the Wandzura-Wilczek approximation [7], in which the contribution from twist-three matrix elements is discarded. The dashed lines indicate the errors on the curves. The errors come almost completely from the error of the nucleon matrix elements.

Unfortunately, our errors on d_2 are yet too large to reach a definite conclusion. But we may say already that the effect of mixing is large, and of the same magnitude as the twist-three contribution itself. At $Q^2 = 5\,\mathrm{GeV}^2$ we now find for the proton

$$\int_0^1 \mathrm{d}x x^2 g_2(x, Q^2) = 0.007(15), \tag{14}$$

and for the neutron we obtain

$$\int_0^1 \mathrm{d}x x^2 g_2(x, Q^2) = 0.006(7), \tag{15}$$

which is not inconsistent with the experimental values [8]. In case of the proton it appears that d_2 vanishes at larger values of Q^2, leaving us with the Wandzura-Wilczek result.

We hope to return with more precise numbers in the near future.

Acknowledgment

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References

[1] R. L. Jaffe, Comm. Nucl. Part. Phys. 19 (1990) 239; R. L. Jaffe and X. Ji, Phys. Rev. D43 (1991) 724.

[2] M. Göckeler, R. Horsley, E.-M. Ilgenfritz, H. Perlt, P. Rakow, G. Schierholz and A. Schiller, Phys. Rev. D53 (1996) 2317.

[3] M. Göckeler, R. Horsley, E.-M. Ilgenfritz, H. Perlt, P. Rakow, G. Schierholz and A. Schiller, Nucl. Phys. B472 (1996) 309.

[4] M. Göckeler, R. Horsley, H. Oelrich, H. Perlt, D. Petters, P. Rakow, A. Schäfer and G. Schierholz, Nucl. Phys. B544 (1999) 699.

[5] M. Baake, B. Gemünden and R. Oedingen, J. Math. Phys. 23 (1982) 944; J. Mandula, G. Zweig and J. Govaerts, Nucl. Phys. B228 (1983) 109.

[6] J. Kodaira, S. Matsuda, T. Muta, T. Uematsu and K. Sasaki, Phys. Rev D20 (1979) 627; R. Mertig and W. L. van Neerven, Z. Phys. C60 (1993) 489.

[7] S. Wandzura and F. Wilczek, Phys. Lett. B72 (1977) 195.

[8] K. Abe et al., Phys. Rev. D58 (1998) 112003; P. L. Anthony et al., `hep-ex/9901006`.